One-dimensional discrete computer model of the subduction erosion phenomenon (plate tectonics process)

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Contents

List of Slides

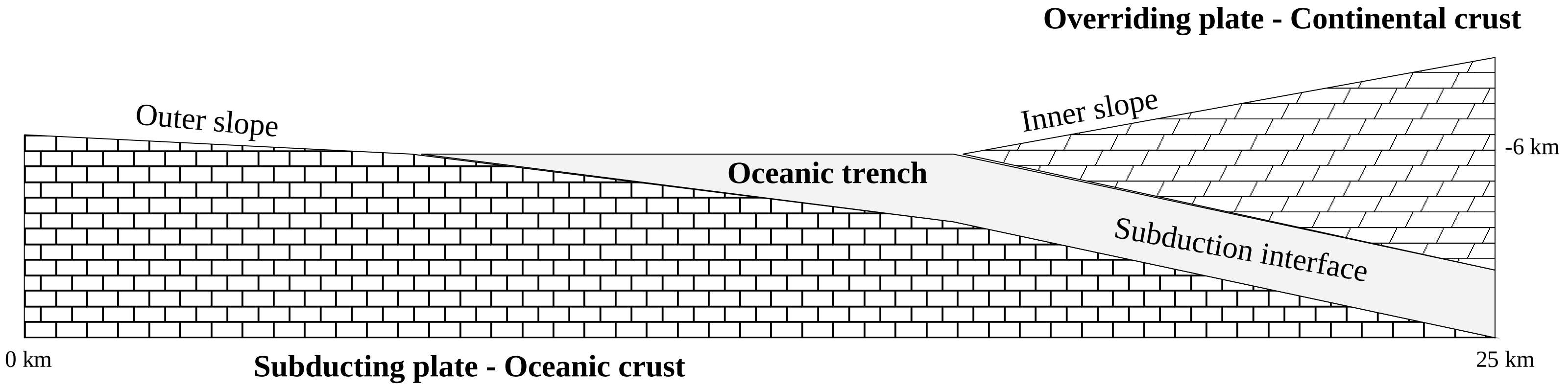
Introduction

- results from a cooperation with the *Laboratoire de Géodynamique Tectonique et Environnement* (Paris),
- refers to a modeling of a geotectonics phenomenon of subduction erosion at the interface between two convergent margins (= tectonic plates),
- an original approach based on Cellular Automata overlap (”an alternative to differential equations in modeling physics”),
- the goal of such studies is to estimate the amount of solid mass of sediments subducted at convergent ocean margins, that is not frontally accreted.

- Up to now, some experimental Sandbox modeling and some global numerical models (Finite Elements methods) have already been made. The goal of such studies is to estimate the amount of solid mass of sediment subducted at convergent ocean margins, that is not frontally accreted.

Subduction of oceanic crust beneath continent

- ▶ The lithosphere of the globe is made of rigid plates of a hundred kilometers' thickness, floating upon viscous asthenosphere,
- ▶ there are relative plates motions of approximately 10 cm a year due to the convection process in the mantle,
- ▶ types of plate boundaries : divergent boundaries (ocean ridges), conservative boundaries (transform faults and fracture zones) and convergent boundaries (trenches)
- ▶ we focus on the case of convergent margins (or plates) in subduction with extension,
- ▶ at convergent margins, the oceanic plate (the lower plate) is subducted beneath a continental crust (the upper plate),



- the extension results from a phenomenon of erosion of the basis of the upper plate by hydrofracturation. All incoming sediment is subducted beneath the upper plate and sink to the viscous asthenosphere.

- The lithosphere (= the outermost[1] layer[2] of the earth, including the crust, reacts as a brittle[3] solid) of the globe is made of a mosaic of rigid plates, which range from 50 to over 200 km thick, floating upon viscous asthenosphere (= a weaker region in the mantle, extending from the base of the lithosphere to the 660 km discontinuity),

- there are relative plates motions due to the convection process in the mantle. Estimated rates of plate velocities range from 1 to 20 cm a year, averaging a few centimeters per year for most plates.

  About plate driving forces : most investigators agree that plate motions must be relative to thermal convection in the mantle. Computer models, however, indicate that plates move in response to slab-pull forces (amount to about 95% of the total driving force), and ridge push and drag forces at the base of the plate (amount to about 5% of the total driving force).

  About convection process : the Rayleigh-Bernard type of convection arises because of heating at the base and cooling at the surface of fluid (convective behaviour of a substance is dependent on a dimensionless number know as the Rayleigh number ;

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[1] le plus à l'extérieur

[2] couche, strate…

[3] cassant

irregular turbulent convection probably exists in the Earth where this number is greater than 10^6) ;

- the boundaries of plates can take three forms : mid-oceanic ridge (where plates are diverging ; accretive or constructive plate margins), trenches (where plates are converging ; destructive plate margins) and transform faults (relative motions of adjacent plates are tangential ; conservative plate margins),
- at convergent margins, the oceanic plate (the lower plate) is subducted beneath a continental crust (the upper plate). In this kind of kinematics, two types of major processes can occur : a compression (type 1 margins, accreting process) and an extension (type 2 margins, subduction erosion process). At type 1 margins, accreting ones, sedimentary material that has been laid down the outer slope, is progressively accumulated to the front of the inner slope. At type 2 margins (21,000 km in global length, all over the world), all incoming sediment is subducted beneath the upper plate and sunk to the viscous asthenosphere.
- The subduction zone is characterized by an oceanic trench, an overpressure of fluids along the subduction interface, a landward dipping zone of earthquakes (Figure 9)...

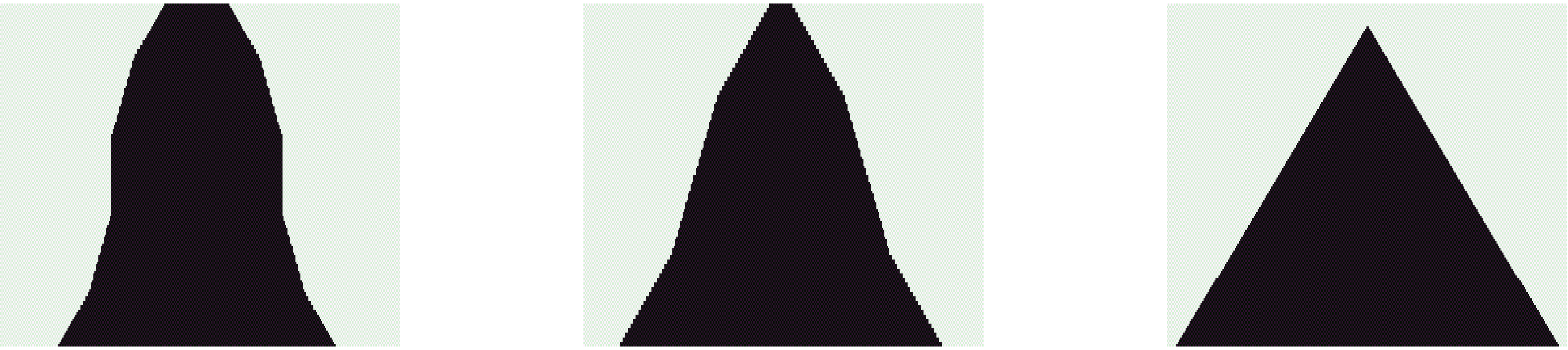
- the extension results from a phenomenon of erosion of the basis of the upper plate by hydrofracturation. All incoming sediment is subducted beneath the upper plate and sink to the viscous asthenosphere.

1D Sand pile model

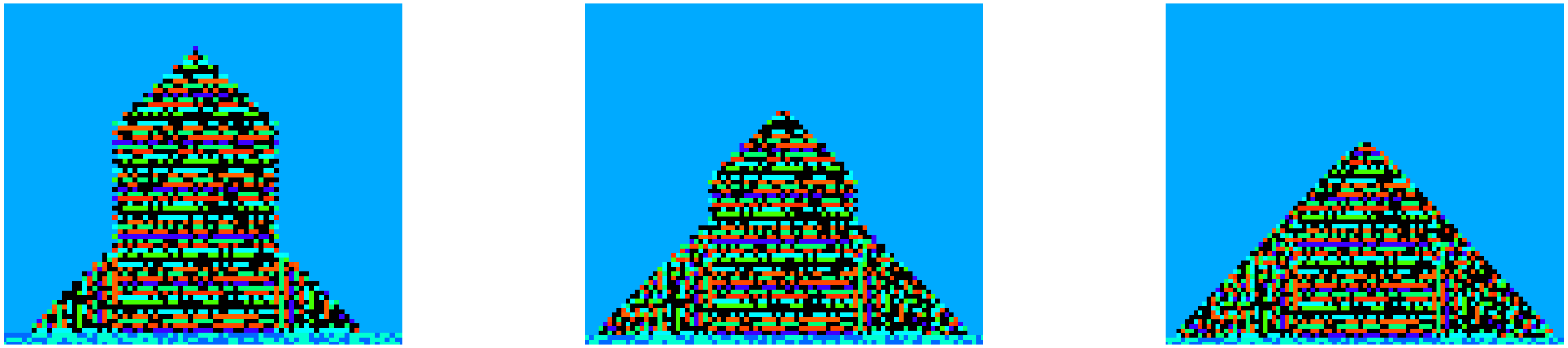
- one of the most interesting phenomena in the dynamics of granular material : the evolution of a pile of granular material (Bak, Tang, Wiesenfeld studied a model based on 1D cellular automaton : the Sand Pile Model),
- a 1D SPM consists of an infinite sequence of stacks (or sizes of stacks). Each stack holds a finite number of grains,
- transition rule of the 1D SPM : let $\mathbb{I}(n) = \begin{cases} 0 \text{ if } n \geq 2 \\ 1 \text{ otherwise} \end{cases}$

$$C_j^{t+1} = C_j^t - \mathbb{I}(C_j^t - C_{j-1}^t) - \mathbb{I}(C_j^t - C_{j+1}^t) + \mathbb{I}(C_{j-1}^t - C_j^t) + \mathbb{I}(C_{j+1}^t - C_j^t)$$

- Screen dumps of the 1D sand pile model we used :



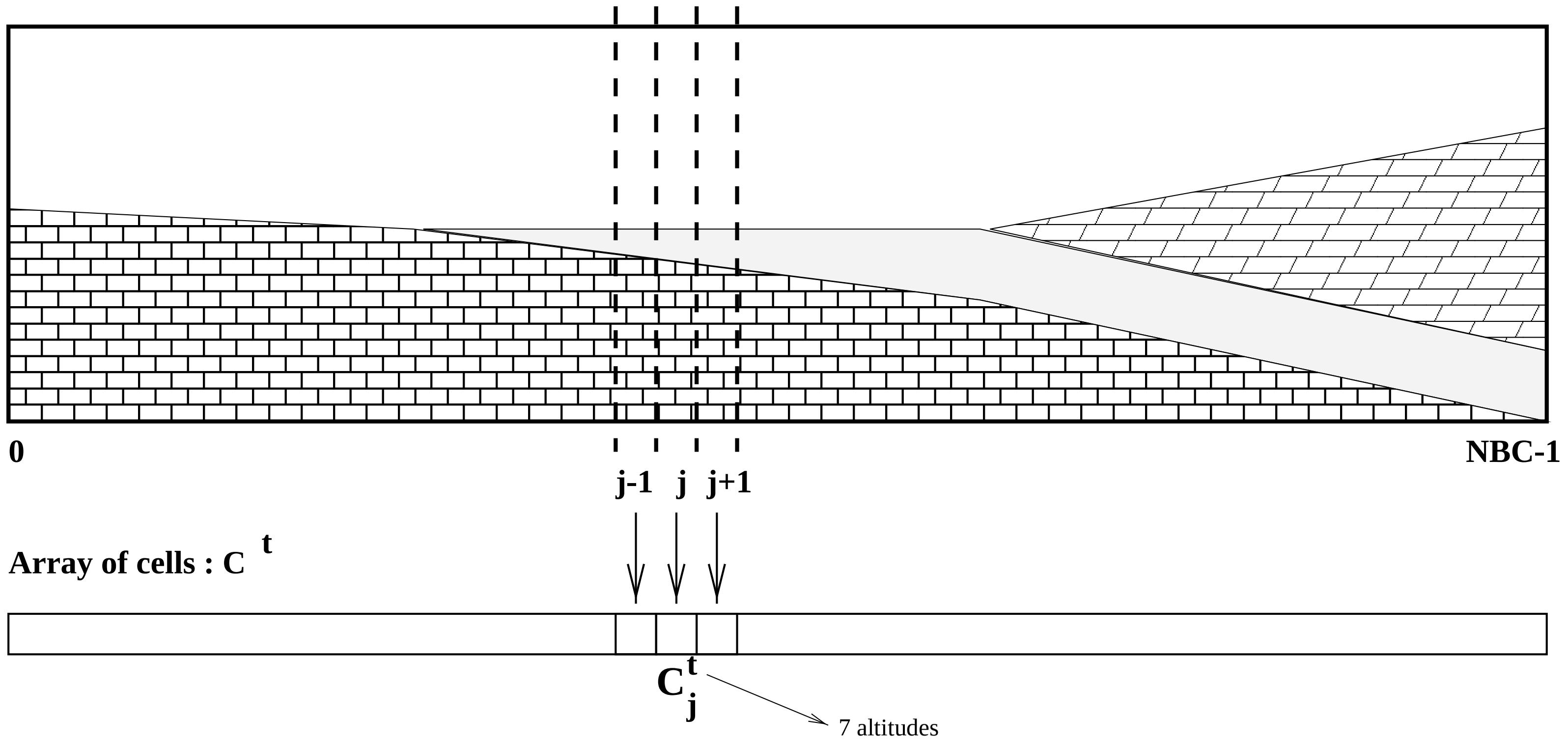
- Screen dumps of the 2D sand pile model we used :



- a 1D SPM consists of an infinite sequence of stacks (more precisely : sequence of sizes of stacks). Each stack holds a finite number of grains. The total number of grains never changes ;
- in SPM, if a stack has at least 2 more grains than its right (or its left) neighbor, then a grain ”tumbles down” from the first stack to its right (or its left) neighbor.
- formally, a cellular automata has three characteristics:
    1. **Parallelism** Individual cells are updated simultaneously and synchronously.
    2. **Locality** The new status of each cell is determined from its position in the grid, by the examining the status of its neighboring cells. The new value of a cell is exclusively based on its old value and the old values of the surrounding cells.
    3. **Homogeneity** All cells of a given type use the same set of rules for updating their status.
- You can think of a cellular automata as a grid built from state machines that react to the activities of their neighbors.

1D model topology

- a discrete universe represented by an array of sites (about 1000 cells),
- a cell is a vertical portion of ”space”,



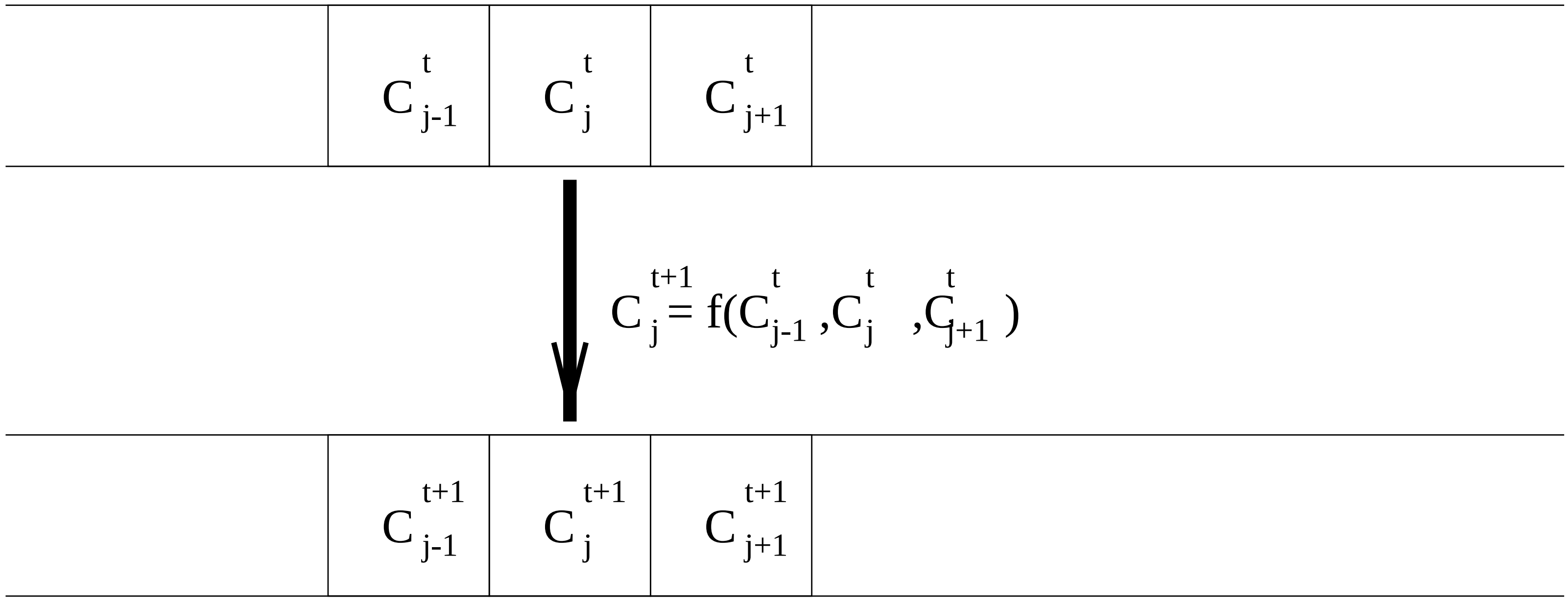
- each cell take a value from a finite set of states,
- a finite linear set (in linear space) of finite automata,

1D CAs are generally easier to handle (there is a much smaller set of possible rules, compared to 2D CAs, according to the fact the neighbourhood of the cell is low). That’s why, as it is simpler to simulate avalanches and self-organized criticality in a sandpile with a one-dimensional cellular automaton rather than with a two-dimensional one, we have first studied a one-dimensional model of the *subduction-erosion phenomenon*.

- a discrete universe represented by an array of sites,
- each cell represents a vertical portion of ”space”, the state of which belongs to a finite sequence. Indeed, the state of a cell is determined by a set of seven cross-section thicknesses. Each of those thicknesses is coded by an integer inferior to the global height.
- therefore, we consider a finite set, in one-dimensional space, of finite automata (cells).

Dynamic of the discrete system

- Starting from an initial configuration, the set of cells evolves at each discrete time step according to a local transition (updating) rule. The sampling of the motion is periodic and the update is local, parallel and synchronous,



$$C_j^{t+1} = f(C_{j-1}^{t}, C_j^{t}, C_{j+1}^{t})$$

- the rule of transition can be seen as a function whose arguments are the states of the cell itself and its neighboring cells, at the previous time step. The evolution of a cell depends only on a local neighborhood of three cells (running cell included),
- an overlap of 3 global functions of transition. They represent 3 different physical phenomena, themselves, on 3 quite distinct scales of time : a dive of the oceanic plate, a subcrustal erosion as well as the subsidences which result from it on all the height from the overlapping plate, and, finally, surface avalanches at the top of the higher plate.

- Starting from an initial configuration, the set of cells evolves at each discrete time step according to a local transition (updating) rule. The sampling of the motion is periodic and the update is local, parallel and synchronous. The rule of transition can be seen as a function whose arguments are the states of the cell itself and its neighboring cells, at the previous time step. The evolution of a cell depends only on a local neighborhood of 3 cells (running cell included).

- To simulate the temporal dimension, we use two arrays of cells : C and CC. Thus, at each iteration (or time step), there is a logical copy of the contents of C to CC. C is the last that has been updated at the previous time step using the total transition rule. C will then be updated again by application of the same transition function, and so on.

- About our automaton : The subducted lithosphere is mechanically smooth and passes from an almost horizontal level in front of the pit to a constant dip under the volcanic arc (beyond 50-100 km). It is generally estimated that the subduction speed is a constant.

- With regard to the boundaries problem, it is necessary to compute separately the cells located at the two limits of the unidimensional automaton (the first introduces material into the system, whereas the last makes possible to eliminate it).

- The engine of ”our” basal erosion is not the subduction of asperities (such as the underwater mounts, mid-oceanic ridge, grabens. . .) on the oceanic plate. It is rather necessary to take into account overpressures of fluid at the top of the subduction interface, which can induce a hydrofracturation.

- Our cellular automaton is, in fact, an overlap of 3 global functions of transition. These 3 functions of transition can be regarded as generalizations of the one-dimensional Sand Pile Model. They represent 3 different physical phenomena, themselves, on 3 quite distinct scales of time. In the circumstances : a dive of the oceanic plate (oblique downward translation under the constraint of the weighty effort exerted by the upper plate), a subcrustal erosion (by hydro-fracturing and overpressures on the outline level of subduction) as well as the subsidences which result from it on all the height from the overlapping plate, and, finally, surface avalanches at the top of the higher plate.

- It is important to distinguish 3 different scales of time. That of the ”instantaneous” phenomena (relative to the other phenomena at least) initially, which occur on a ”fast” scale of time (it is the case of the surface avalanches). That of the phenomena ”with great width”, then, which occur on a ”slow” time scale (it is the case of the subduction even of the oceanic margin with the ”generation of steps” which is correlated to it, but it is also the case of ”ageing” and the subcrustal erosion of the overlapping continental margin).

- Last, we introduce an ”average” scale of time to represent physical phenomena intercalating themselves between the two classes of above mentioned phenomena (it is the case of the general translation of the face of the oceanic margin, of the levelling of the oceanic trench and the subsidences within the overlapping margin).

A parallel implementation on the CRAY T3E computer

- development of a new dedicated software,
- parallel strategy : work and data are distributed among the processor elements,
- a simple and natural one-dimensional domain decomposition,
- after each iteration, solution values on the boundaries of a subdomain need to be exchanged with the adjacent subdomain $\Rightarrow$ each processor will exchange messages with its left and right neighbors,
- the tandem C and PVM library for questions of portability.

- There already exists, in the public domain, software tools making possible to visualize or handle cellular automata. However, it seems to us necessary to develop our own platform. Indeed, although very effective and very useful for a two-dimensional simulation, *The Cellular Automata Simulation System*, developed around the Cellang language, cannot absolutely be appropriate to us here, since it does not allow us a two-dimensional visualization in the case of a uni-dimensional automaton.

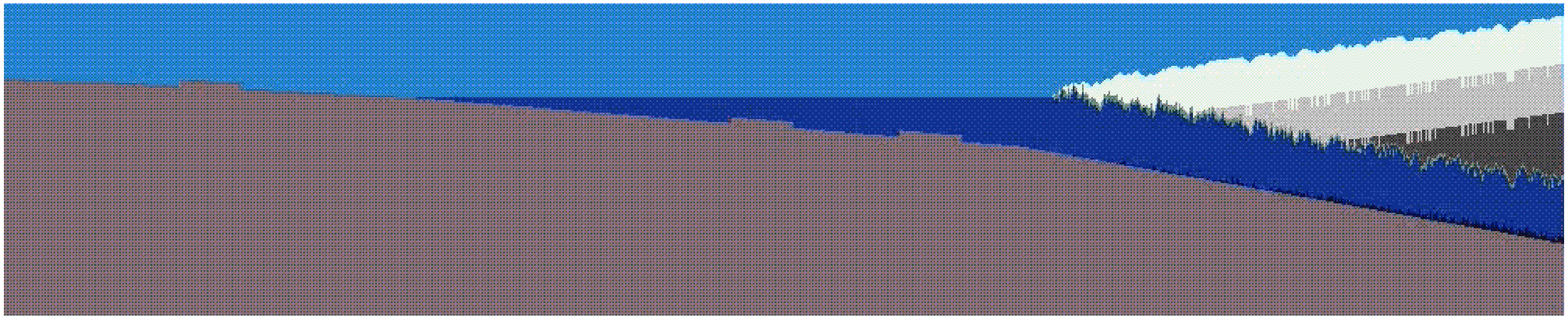
- The parallel strategy we use is the following one : work and data are distributed among the processor elements. We use a simple and natural one-dimensional domain decomposition : the array of cells has been divided into a number of parts (subdomains of contiguous cells) equal to the number of PEs. After each iteration, solution values on the boundaries of a subdomain need to be exchanged with the adjacent subdomain. We can see that each processor will exchange messages with its left and right neighbors except the first and last ones which have respectively no predecessor and no successor. That’s why there is an increase of communication time as the number of processors increases.

- We chose the tandem C and Parallel Virtual Machine library for questions of portability. Thus our application functions as well, except for the performances, on a network of heterogeneous workstations as on a Massively Parallel Processors computer like the CRAY T3E (concerning the processing and the first graphic post-processing at least).

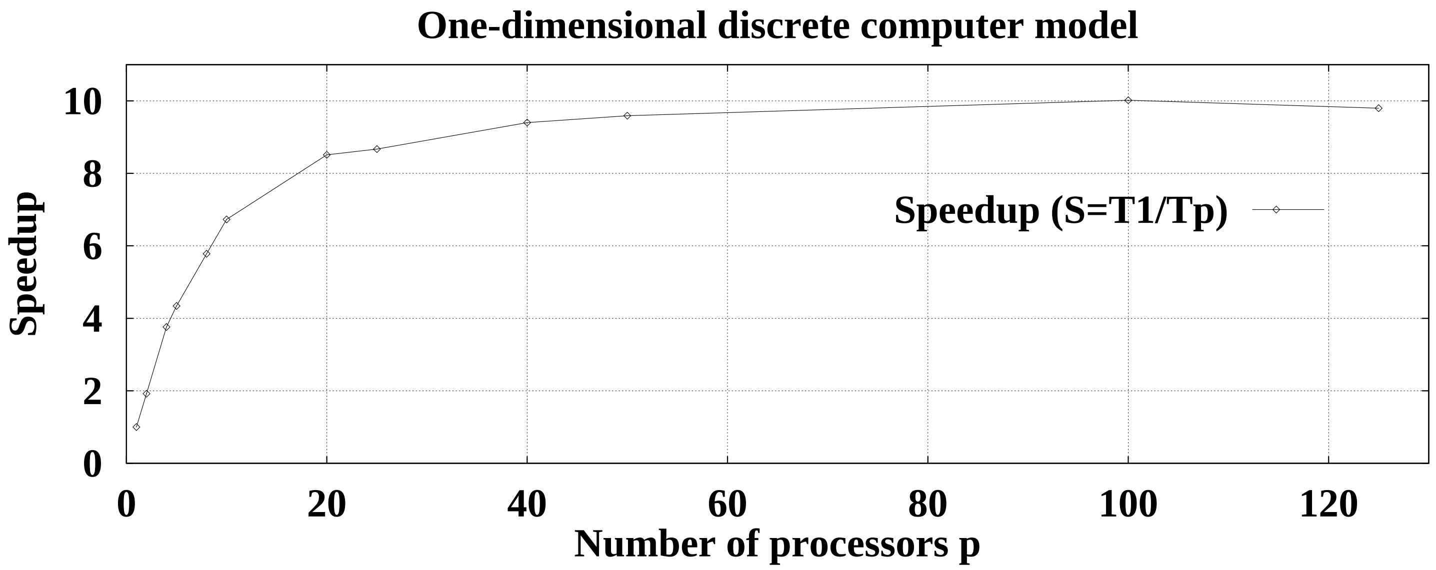
Results of the one-dimensional simulation

- MPEG motion pictures have been realized using the *convert* tool of the *ImageMagick* software on a four-processors Silicon Graphics computer. These images were generated, for the processing, on a CRAY T3E with 256 processors DEC ALPHA EV5, and, for post processing, on a four-processors Silicon Graphics Power Challenge XL.

- visual rendering of the one-dimensional simulation :



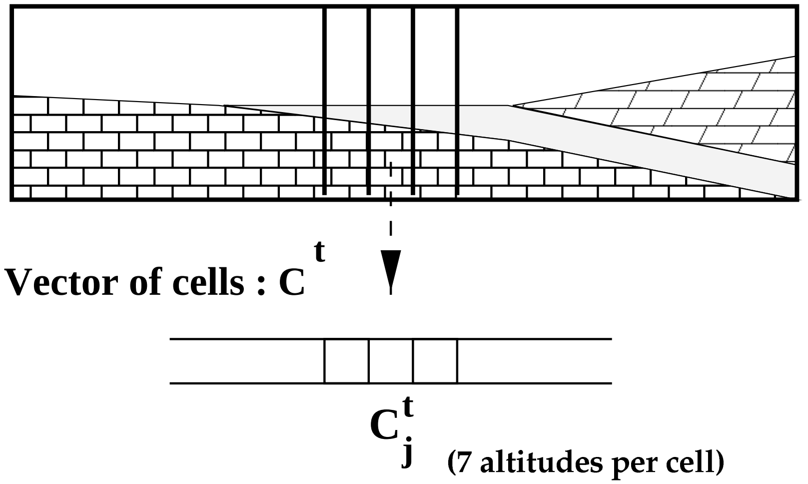
- in order to optimize the performance of the processing, we have overlapped time of communications by time of computations (non-blocking sending of boundaries values, computation of local data, receiving of updated values from the neighboring PEs, last computations of boundaries cells),


One-dimensional discrete computer model

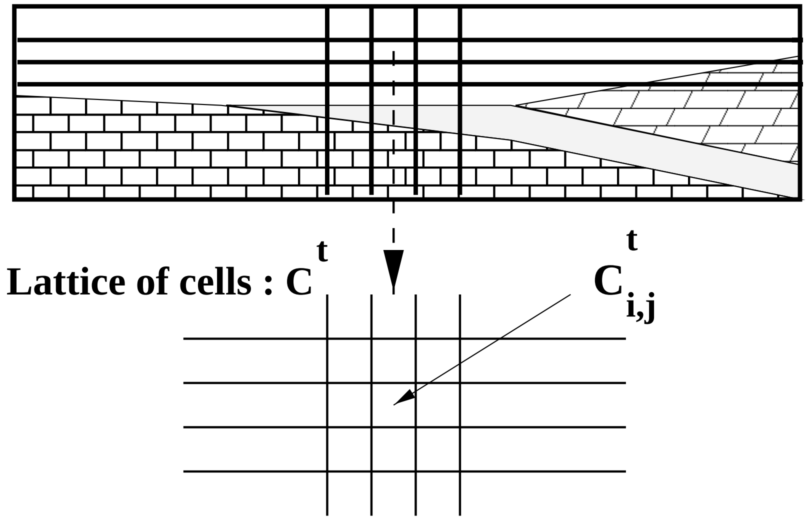
- Our local transition function, is able to represent phenomena on a wide time scale such as : a dip of the lower subducted plate (80 km / 1 million years), a landward migration of the front of the upper plate and the trench axis (20 km / 1 million years), a subcrustally erosion of the continental margin basement, an underthrust of sediment material and avalanches at the top of the overriding plate (almost instantaneous phenomena compared with the dip or the landward migration).
- As shown in the figure, the dynamical behavior of our model is proper. We are expecting from a further improvement of the two dimensional cellular automaton we are developing, that it will also represent a frontal accretion, a subcrustally underplating and an internal fluid overpressure causing hydrofracturing effects in the upper plate.

From 1D model to 2D model

One-dimensional model



Two-dimensional model

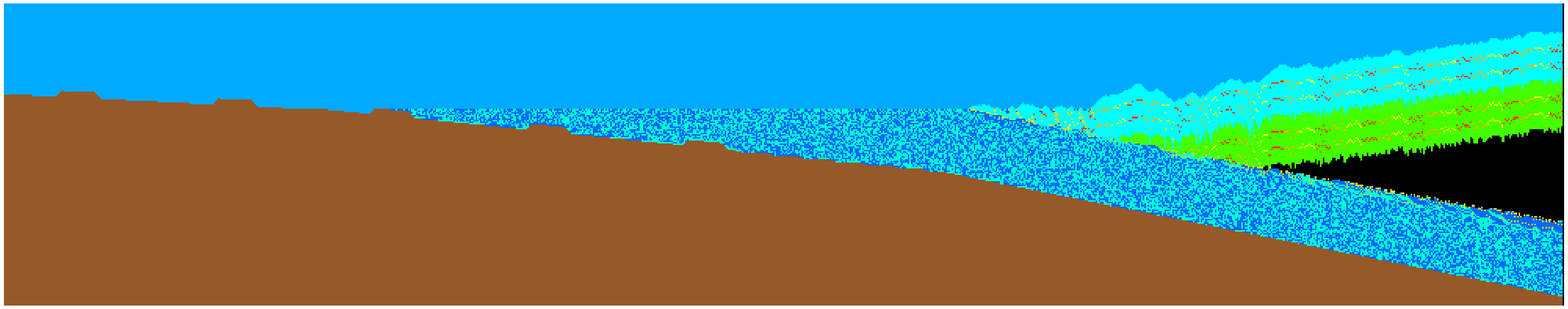


One-dimensional discretization of space (one-dimensional model) in comparison with the two-dimensional discretization of space (two-dimensional model).

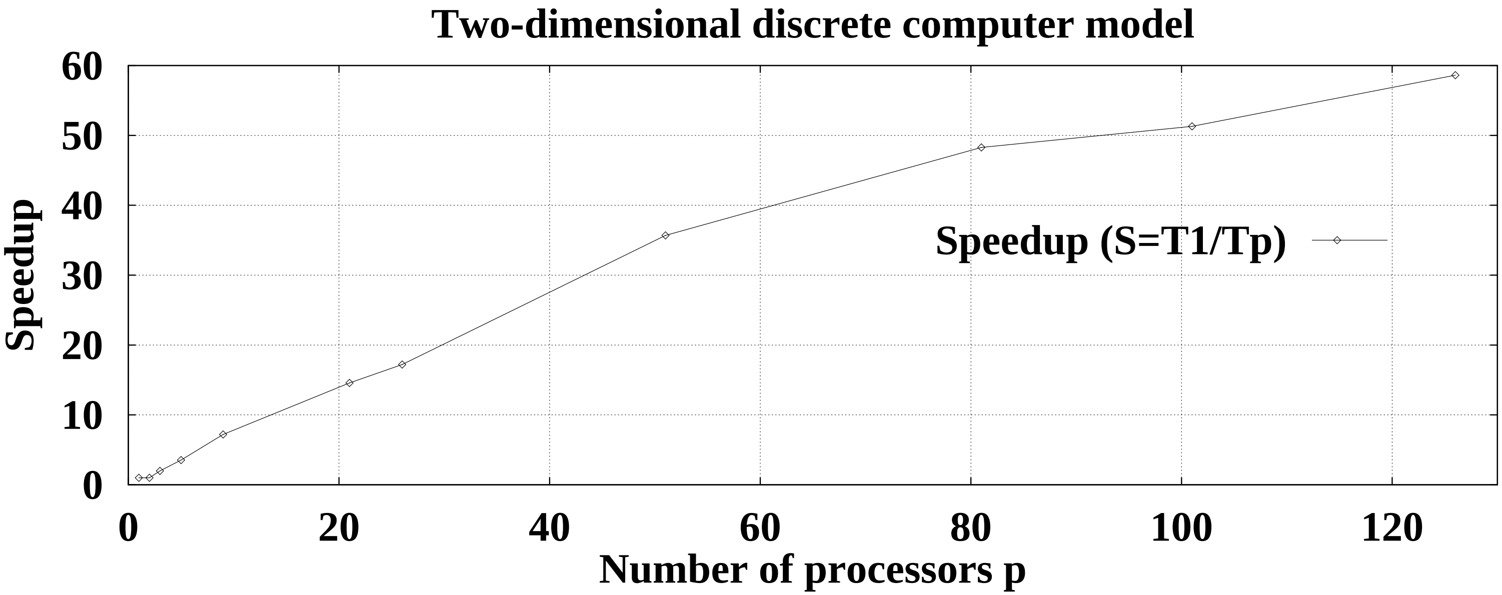
- 1D model : a discrete universe represented by an array of sites (cells). Each cell represents a vertical portion of ”space”, the state of a cell is determined by a set of seven cross-section thicknesses.
- advantage of this model : the amount of information to be treated is weak
- 2D model, the discrete system consists of a two-dimensional lattice of 200,000 cells. Cells updating uses a local transition function, only depending on the previous states of 25 neighboring cells (running cell, contiguous cells and contiguous cells of contiguous cells included).

Results of the two-dimensional simulation

- MPEG motion pictures have been realized using the *convert* tool of the *ImageMagick* software on a four-processors Silicon Graphics computer. These images were generated, for the processing, on a CRAY T3E with 256 processors DEC ALPHA EV5, and, for post processing, on a four-processors Silicon Graphics Power Challenge XL.
- visual rendering of the two-dimensional simulation :



- Even though mapping of 2D-tori theoretically is a simple problem, the MPI library *virtual topology* mechanism (or logical process arrangement) seems to be a convenient process naming structure ⇒ we have decided to use the MPI library.



- Theoretically, none of the cells of the lattice has knowledge of its position (that is to say of its coordinates) in the matrix of cells. Furthermore, cells located at the boundaries of the finite two-dimensional lattice have also to communicate with their own neighbors. So the discrete system has to be mapped by a two-dimensional fully periodic grid of cells (a 2D-torus) ;
- the tests were executed upon a CRAY T3E and a network of Linux 2.0 workstations (using the LAM 6.1 implementation of MPI).
- the speedup of this two dimensional simulation is almost linear and increasing. We obtained an optimum speedup of about 60 ;
- in the two dimensional case, the interest of a parallel solution is obvious, since the simulation time in the parallel case is of about 42 seconds, instead of 2,462 seconds (about 40 minutes) in the sequential case.
- [1, 3, 4, 2]

Conclusion and future developments

- ▶ development of the *LAC platform*. This platform consists of :
  - a programming language LAC[a],
  - an associated compiler that generates C-code using MPI message passing library,
  - a Graphical User Interface[b].

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[a] Language pour Automates Cellulaires

[b] Written using the interpreted scripting language Perl/TK. . .

References

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Web sites

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- http://penguin.phy.bnl.gov/www/xtoys.html